Day 5 – Right Triangle Trigonometry

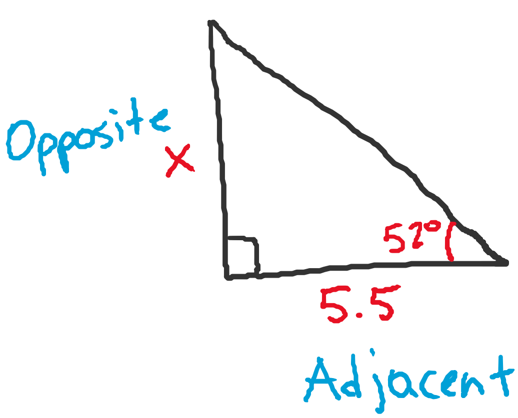
Task 1. The solar collector in the picture to the right functions at maximum efficiency when the angle of elevation formed with the level ground is approximately 52°, and it currently is set at that angle. The distance along the ground from the back legs to the front legs is 5.5 feet.

A. At what height are the back legs currently set? (Round to the nearest inch)

B. How long is the face of the collector?

C. If the back legs were to be lowered by one foot and pushed back, how would the angle of elevation change?

A. The side of this solar collector has the shape of a right triangle. Let's draw a picture to help us:



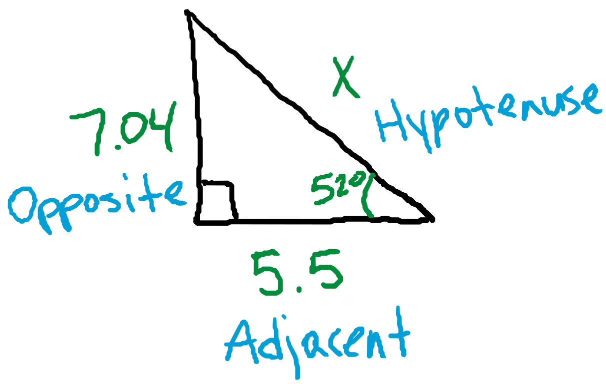
To solve this problem using the information given to us, we need to use SOH CAH TOA.

Since we are working with opposite and adjacent, remember, we use tangent.

Angle $\tan 52 = \dfrac{x}{5.5}$ $\quad \dfrac{\text{Opposite}}{\text{Adjacent}}$

$(5.5)(\tan 52) = x \Rightarrow \boxed{x = 7.04 \text{ feet}}$

B. Let's re-draw our triangle.



The face of the collector is the diagonal of the triangle. To solve this problem, we could use SOH CAH TOA or $a^2+b^2=c^2$. (Pythagorean's Theorem)

I'll use CAH (cosine, adjacent, hypotenuse).

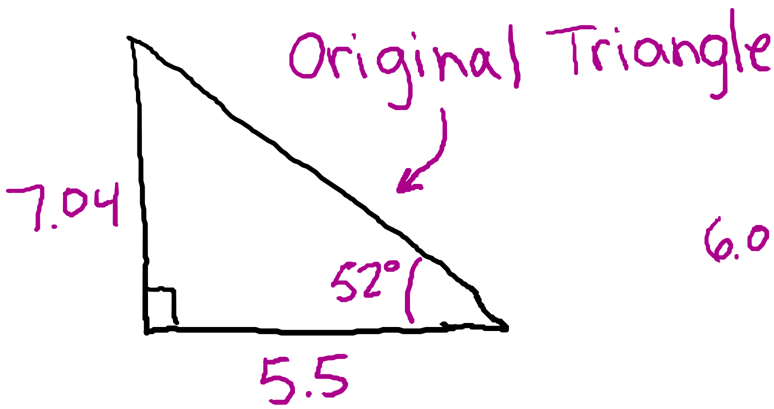
$$\cos 52 = \frac{5.5}{x}$$

Remember our short cut of switching cos 52 and x since the x is in the denominator.

$$x = \frac{5.5}{\cos 52}$$

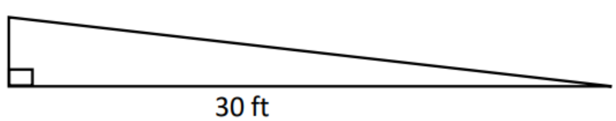
$$\boxed{x = 8.93 \text{ feet}}$$

C. This is a logical question to just think about. If the back legs are shortened and pushed back from the front, this means the angle of elevation will have to be smaller.



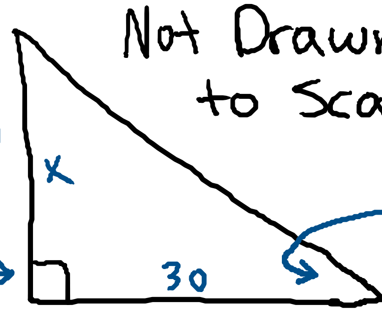
Pushed Back (Meaning Longer than 5.5 Feet)

Task 2. You are building a house with a ramp. The Americans with Disabilities Act (ADA) limits the longest single span of ramp prior to a turn platform to 30 feet. If you don't have any turns in your ramp, what do you believe is the maximum height the base of the door could be?



This is one of those silly, unrealistic problems that we are asked sometimes.

Since this angle is 90°, the other two angles must add to equal 90°.

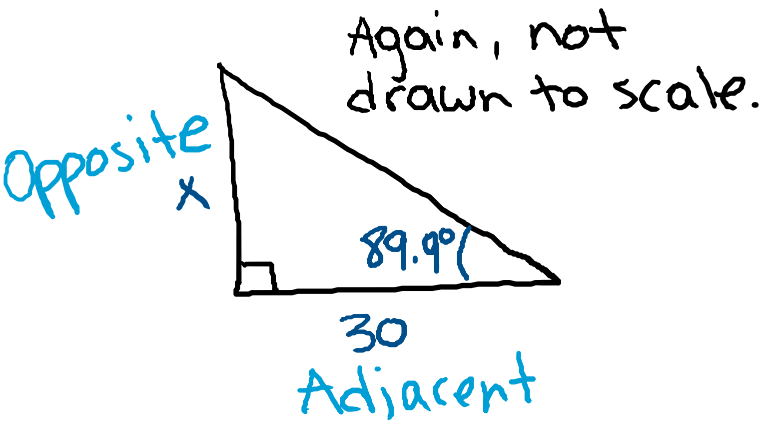


Not Drawn to Scale

Now, to make x the maximum it can be, we need to make this angle the maximum it can be.

Recall, and use your arms to try different angles and see, that the larger the angle, then the larger the side the angle opens up to.

So, the largest number below 90° is $89.\bar{9}$ (but let's just use 89.9 as an approximation).



Let's use tangent to solve for x.

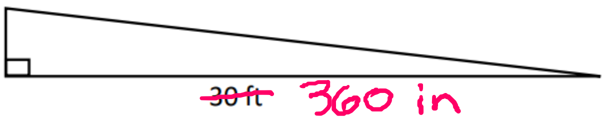
$$\tan 89.9 = \frac{x}{30}$$

$$(30)(\tan 89.9) = x$$

$$\boxed{x \approx 17{,}188.7 \text{ feet}}$$

Again, unrealistic, but the base of the door could be approximately this tall and the triangle (or ramp) still work as a right triangle.

B. Let's figure out what the actual angle is. According to the ADA, there must be 12 inches of length (run) for each inch of rise. Knowing that, what must the angle of elevation be?



This problem is more practical. The main thing to see is 12 <u>inches</u> and 30 <u>feet</u>.

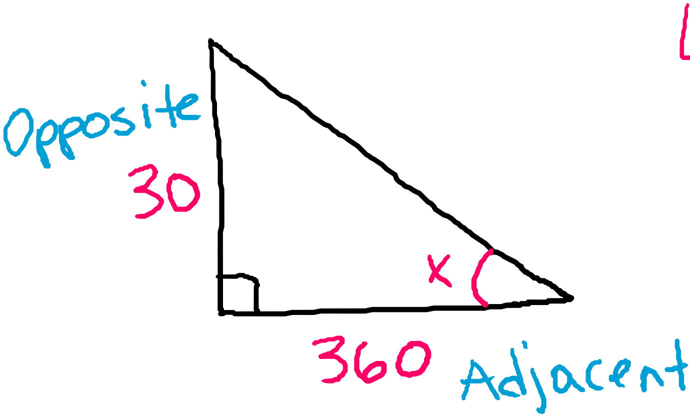
So, let's convert 30 feet into inches:

12 inches in 1 foot $\Rightarrow 30 \cdot 12 = 360$

Next (and it seems we are using the same numbers because of the problem), we need to determine the rise (vertical height) of the ramp.

1 inch of rise for every 12 inches of run.

$$\frac{360 \text{ inches}}{12 \text{ inches}} = 30 \text{ inches of rise}$$



Let's use tangent:

$$\tan x = \frac{30}{360}$$

$$x = \tan^{-1}\left(\frac{30}{360}\right)$$

* You could have converted to feet instead of inches and got same answer.

$$\boxed{x \approx 4.8^\circ}$$