Day 3 – Quadratic Equations and Inequalities

Task 1

Suppose $h(t) = -5t^2 + 10t + 3$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard.

1. How high above the water is the springboard? Explain how you know.
    a. 2. Approximately, when does the diver hit the water? Can you calculate this multiple ways?
    b. 3. During what time span on the diver's descent toward the water is the diver below the springboard?
4. When does the diver reach the peak of the dive? What was the maximum height?

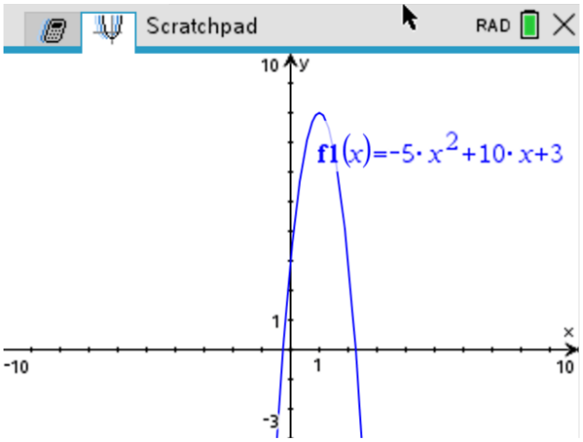
1. h(t) is written in Standard Form, which which tells us the y-intercept. Remember, the y-intercept is our starting (or initial) point. So, because the y-intercept is (0, 3)... the springboard is 3 meters above the water.

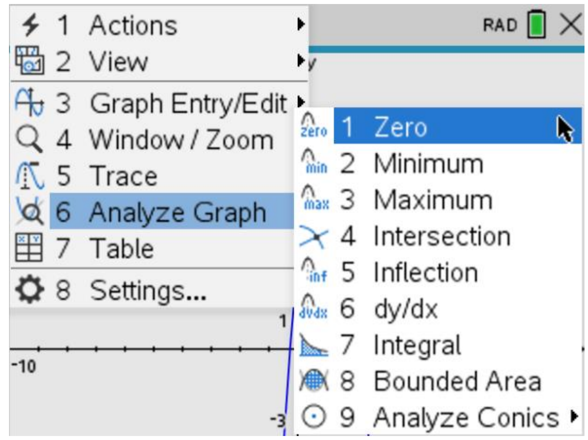
2. Approximately, when does the diver hit the water? Can you calculate this multiple ways?

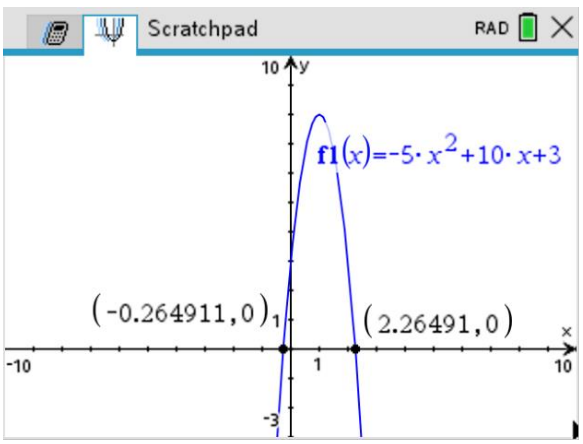
2. There are 3 ways for us to find the x-intercepts (which represents when the diver hits the water):
- Factoring (we cannot this problem)
- Graphing or Using a Table
- Quadratic Formula

$\longrightarrow \quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$h(t) = -5t^2 + 10t + 3$







Graphically, by finding the zeros, we see the graph crosses the x-axis at (-0.265, 0) and (2.265, 0).

Extraneous Solution (Negative Time)

Now, to use Quadratic Formula:

$$x = \frac{-10 \pm \sqrt{10^2 - 4(-5)(3)}}{2(-5)} \longrightarrow 100 + 60 = 160$$

$$= \frac{-10 \pm \sqrt{160}}{-10} \longrightarrow \sqrt{160} \approx 12.64911$$

$$= \frac{-10 \pm 12.64911}{-10}$$ → Remember to split into 2 different problems.

$$= \frac{-10 + 12.64911}{-10} \qquad = \frac{-10 - 12.64911}{-10}$$

$$= \frac{2.64911}{-10} \qquad = \frac{-22.64911}{-10}$$

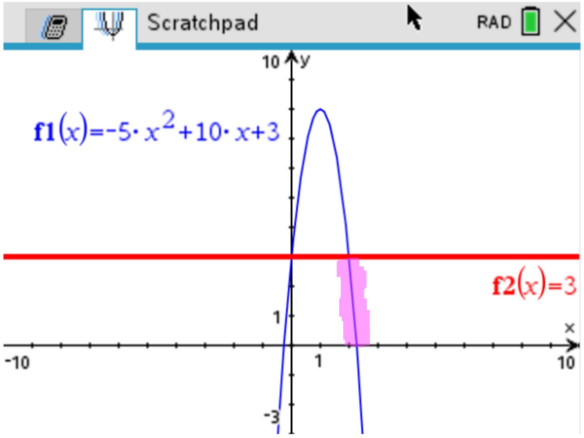
$$\approx -0.265 \qquad \approx 2.265$$

Extraneous Solution (Negative Time)

$$\boxed{\text{So, by graphing or quadratic formula, we determine the diver hits the water at } t \approx 2.265 \text{ seconds.}}$$

3. During what time span on the diver's descent toward the water is the diver below the springboard?

3. Firstly, the word descent means going down or decreasing. Let's look at a graph for a visual representation:



For this problem, we want to know the highlighted region when the diver is below the line $y=3$.

To solve this, we need to set the quadratic equation of our diver equal to 3.

$$-5t^2+10t+3=3$$
$$\underline{\quad -3 \quad -3}$$

Solve by getting it equal to 0.

$$\frac{-5t^2}{-5t}+\frac{10t}{-5t}=0$$

We could graph or use quadratic formula, but it's easy to GCF.

$$-5t(t-2)=0$$

$$-5t=0 \qquad t-2=0$$

$$t=0 \qquad t=2$$

Set each factor equal to 0, and yes, the GCF is a factor.

The diver is at the height of the springboard at $t=0$ (makes sense since that's where the diver starts) and $t=2$. We are only concerned with $t=2$ since that is within the diver's descent.

Also, recall from #2, the diver hits the water at $t=2.265$ (which is when the time stops).

$\boxed{\text{So, the diver is below the springboard when } 2 < t \le 2.265 \text{ seconds.}}$

4. When does the diver reach the peak of the dive? What was the maximum height?

This problem wants to know the maximum point (also called the vertex).

Let's use the formula $x = \frac{-b}{2a}$ to determine the x of the vertex (or AOS).

$h(t) = -5t^2 + 10t + 3$

$t = \frac{-10}{2(-5)} = \frac{-10}{-10} = 1$

Represents the x of the vertex and the axis of symmetry.

Vertex (Max Point): $(1, 8)$

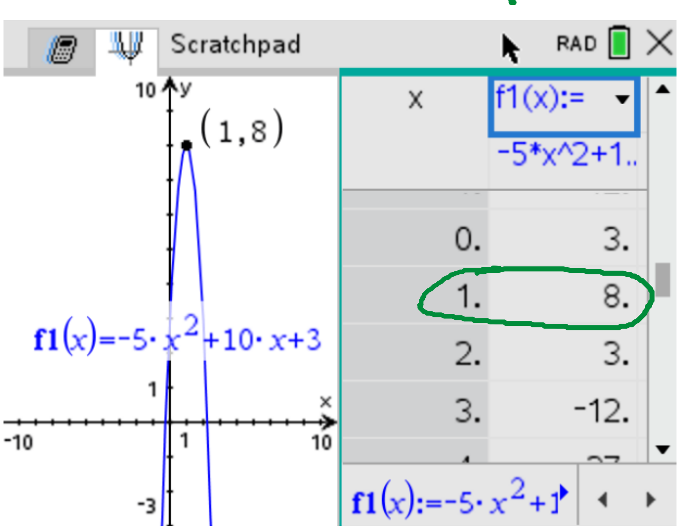
$h(1) = -5(1)^2 + 10(1) + 3$

$= -5 + 10 + 3$

$h(1) = 8$

To find the y, substitute $t = 1$ into our quadratic function.

We can also use a graph to find the vertex point:



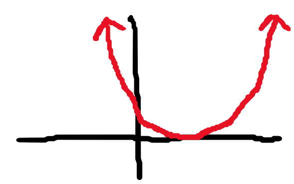
The diver reaches the maximum height of 8 meters at 1 second after the jump.

Task 2

Directions: Using the digits 0 to 9 at most one time each, fill in the boxes to so that there is only one solution.

$$\square x^2 + \square x + \square = 0$$

Remember, a quadratic has only one solution when its graph touches the x-axis only one time.



Now, also remember, for this to happen, the quadratic's factors must be the same... one solution is also called a double root.

So for instance, $y=(x+10)(x+10)$ which can be rewritten as $y=(x+10)^2$ has only one solution (double root).

This problem wants us to write the equation in Standard Form, which means we need to multiply our Factored Form equation. And, since we can only use numbers 0-9, we need to keep our factor numbers small.

Try $(x+1)(x+1)$

$x^2+x+x+1$

x^2+2x+1

$\boxed{1}x^2 + \boxed{2}x + \boxed{1} = 0$

Does not work since we repeated numbers.

Try $(x+2)(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

$\boxed{1}x^2 + \boxed{4}x + \boxed{4} = 0$

Again, does not work since we repeated numbers.

Try $(x+3)(x+3)$

$x^2+3x+3x+9$

x^2+6x+9

$\boxed{\boxed{1}x^2 + \boxed{6}x + \boxed{9} = 0}$

WORKS!